

A STUDY
OF THE
EFFECTS OF TEMPERATURE
UPON A TUNING FORK

BY
EUGENE CYRUS WOODRUFF

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THESIS PRESENTED TO THE FACULTY OF THE UNIVERSITY OF
MICHIGAN FOR THE DEGREE OF DOCTOR
OF PHILOSOPHY



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BY EUGENE CYRUS WOODRUFF.

THE temperature coefficient of a tuning fork has been frequently investigated during the past fifty years. In most cases, however, the range covered was limited to little more than ordinary temperatures, and the results threw but little light upon either the actual character of the effect of temperature or upon the cause of this effect. In other cases where a wider range of temperature was attempted, the methods employed left something to be desired in convenience or in sensitiveness. With the method used in this research the ease and accuracy of the observations were nearly as great at high temperatures as at the lower ones, and the range of temperature employed left little to be desired, either as concerns the effect on the frequency of the fork or as regards the effect of temperature upon the elasticity of steel.

In the course of the investigation the question of the effect of temperature on the damping of the vibrations of the fork presented itself. Hitherto this phenomenon seems to have been almost wholly overlooked. In this research it came to light as a natural result of the method employed.

Mayer,¹ McLeod and Clarke,² Ellis,³ Shearer,⁴ Michelson,⁵

¹ Amer. J. of Sci. [3], 30, p. 485, 1885.

² Phil. Trans. Roy. Soc., 171, part 1, pp. 1-15, 1880.

³ Nature, 33, p. 54, 1885.

⁴ PHYS. REV., I., pp. 291-296, 1894.

⁵ Amer. J. of Sci. [3], 25, p. 61, 1883.

Mercadier,¹ Kayser,² and König,³ have all found the frequency of a diapason to be a linear function of the temperature. Pierpaoli,⁴ found otherwise. Most of these investigations, however, covered but a small range of temperature, and many distinctly specify that their coefficient applies only within that range. The function is practically linear, as Kayser says, from 0° C., to 30°, but not exactly so, and the departure from linearity becomes more marked with increasing temperature.

The following table shows the results of these investigators:

	Observer.	Frequency of Fork.	Range of Temperature.	Coefficient per 1° C.	Coefficient per 1° F.
1	Mayer.	256			0.000046
2	McLeod and Clarke.	256	15-26°	0.00011	
3	Ellis.				0.000054
4	Shearer.		20-80		
5	Michelson.	128 v. s.	12-24	0.00010	
6	Mercadier.	56 "	3-26	0.000096	
7	Kayser.	144 "	0-26	0.000088	
8	"	144 "	"	0.000105	
9	"	170 "	"	0.000098	
10	"	170 "	"	0.000100	
11	König.	512 "		0.000111	
12	"	512 "	26-56	0.000107	
13	"	128 "	3-26	0.000111	
14	Pierpaoli.	145 "	0-30	0.000097	
15	"	145 "		0.000101	

Mayer used the graphical method. McLeod and Clarke used a stroboscopic method in which the fork was compared directly with a ruled, rotating drum, which was timed by an electric clock. Shearer kept one fork at constant temperature while another was heated electrically and the two were compared by means of Lissajous' figures. The heated fork was alternately bowed and electrically driven. König used his *Stimmungabel-uhr*, running it for hours at a time, at various temperatures, in a room whose temperature was kept constant. Again a fork was heated for several hours in a wooden case with an iron bottom and then compared with the clock-

¹ C. R., 83, pp. 822-824.

² Wied. Ann., N. F., 8, p. 444, 1879.

³ Wied. Ann., N. F., 9, pp. 394-421, 1880.

⁴ Fortschritte, p. 558, 1889.

fork. Kayser compared two forks by means of Lissajous' figures. The forks were heated by surrounding them with hot-water jackets. Michelson compared his fork stroboscopically with a pendulum, using substantially the method described by Rayleigh.¹ A modification of Michelson's method described by Reed,² combines greatly increased accuracy with ease of observation. This method has been used in this investigation.

Wertheim³ found that the coefficient of elasticity of iron and steel increased from -15° to $+100^{\circ}$ and then decreased up to 200° to a value less than that at ordinary temperatures. Kohlrausch and Loomis⁴ investigated the elasticity of steel wire, using torsional vibrations, from 0° to 90° , without finding any suggestion of the approaching maximum reported by Wertheim. Miss Noyes⁵ investigated Young's modulus by stretching, between the temperatures 15° and 180° , quite carefully, and made a few observations even up to 600° . She differs from Wertheim concerning the existence of a point of inflexion in the elasticity curve between 100° and 200° .

Concerning the effect of temperature upon the damping of a tuning fork but two citations can be made.

König⁶ says: "Die bis zur Temperatur des kochenden Wassers erhitzten Gabeln nur noch eine ausserordentlich kurze Schwingungsdauer haben." On account of this he found difficulty in his observations at the higher temperatures.

Mayer⁷ also reports a similar effect of change of temperature on the duration of the motion of a vibrating body. He says the sound from brass, bell-metal, aluminum, steel, zinc, silver and glass falls off in intensity and duration with rise in temperature between 0° and 100° C. Steel that vibrated for 80 seconds at 0° C. vibrated for but 5 seconds at 100° C. However neither König nor Mayer pushed the matter beyond 100° C.

¹ Phil. Trans. Roy. Soc., part I, p. 316, 1883.

² PHYS. REV., p. 279, 1901.

³ Ann. de Chimie et de Physique [3], XII., 1844, pp. 385, 581 and 610.

⁴ Pogg. Ann., CXXI., p. 481, 1870.

⁵ PHYS. REV., p. 277, 1895.

⁶ Wied. Ann., 9, p. 411, 1880.

⁷ American Journal of Science, p. 41, 1896.

The same difficulty presented itself in this investigation. *Moreover the duration of motion was found progressively to decrease with increasing temperature up to a certain point. On raising the temperature still higher, the duration of motion progressively increased until at last it was almost as great as at ordinary temperatures.*

The three points investigated are :

1. The temperature coefficient of a tuning fork between 0° and 200° .
2. The variation of the coefficient of elasticity of steel with rise of temperature.
3. The damping effect of temperature upon the vibrations of a tuning fork.

The thesis falls into three parts : Part one covers the method and apparatus employed ; part two the accumulated data ; and part three a discussion of the results, the curves, and the theoretical deductions.

PART I. METHOD AND APPARATUS.

1. *Method.* — As mentioned above, the method is stroboscopic. But one fork is used and that is compared directly with a free special pendulum. The nearest whole number of vibrations of the fork is determined graphically. The fraction of a vibration plus or minus is then obtained by observing a stroboscopic cycle between fork and pendulum as described by Rayleigh, Michelson and Reed. Some means of securing a cycle of any desired length, whatever the frequency of the fork, is necessary if the temperature, and, therefore, the frequency of the fork is to be varied over any wide range. The free pendulum used was the one described by Reed. The period of the pendulum was varied by placing small weights in a pan carried by the upper ball. Thus, whatever the frequency of the fork, any desired length of cycle was readily secured.

The period of the pendulum was determined by comparing it with the laboratory clock by means of flash coincidences as explained below ; then the fork and pendulum were compared by Reed's method. The equation for the frequency of the fork is

$$N = \frac{n + c}{P} ;$$
 where N is the number of complete vibrations per second of the fork, P is the period of the pendulum, n the nearest in-

tegral number of vibrations in time P , and c the reciprocal of the number of flashes per cycle between the fork and the pendulum. To determine the sign of c , a very small rider was placed in the pendulum pan, thus increasing P slightly but not enough to change the value of n . Now if n remain unchanged and the number of flashes per cycle be observed to increase the stroboscopic motion is a retrograde one and hence c is negative. On the other hand, if the number of flashes per cycle decrease under the same conditions, the motion is progressive and c is positive.

2. *Apparatus.* — The pendulum and its accessories were mounted upon a brick pier. The supporting framework, which was strong and sufficiently rigid for the purpose, was firmly clamped to the stone slab. The bearings of the pendulum consisted of two polished steel planes secured face downward to the under side of the supporting collar. Two steel bicycle balls, about one millimeter in diameter, were set in recesses in copper plates fastened to the framework on either side of the pendulum. On these balls the pendulum planes swung with almost perfect freedom. The total mass of the moving parts was about twenty-five kilos.

Owing to the large mass of the pendulum the two systems of rotating mirrors caused little decay in amplitude during the time required for a complete set of observations. By tuning the pendulum with weights instead of mercury it could be restored to any previously used condition, thus making data regarding the provisional period and other characteristics peculiar to any given condition immediately available. The amplitude was measured by placing immediately below the lower end of the pendulum a paper millimeter scale. A distant incandescent lamp cast a sharp shadow of the edge of this part of the pendulum upon the scale, and the motion of the shadow could be readily observed to tenths of a millimeter.

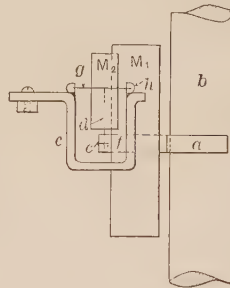


Fig. 1.

Fig. 1 shows the system of mirrors use for timing the pendulum. While the general method followed was that used by Mendenhall,¹

¹ U. S. Coast and Geod. Surv., 1891, part 2, App. 15.

yet the arrangement of the mirrors was changed so as to give much greater sensitiveness, in fact any degree of sensitiveness desired, and the method of illumination was such as to make the process of actual observation largely automatic.

One mirror M_1 , 2.5×7.5 cm., was fastened to an arm a , projecting from the pendulum rod b twenty centimeters below the pendulum bearings. The mirror thus moved through the same angle as the pendulum itself. The mirror was made long so that *all the flashes*, and not merely those near a coincidence, would appear in the telescope. Then a skilled observer could, by observing the character and position of a few flashes, judge what would be the time and character of the next coincidence. The pendulum thus required very little watching, and the operator was free to make observations on the fork between coincidences.

Immediately in front of the mirror M_1 was placed a smaller mirror M_2 , about 1×3 cm. This mirror was supported on horizontal bearings g , h , of fine needle points in glass centers, drawn from thick-walled capillary tubing. These latter were mounted on an adjustable arm c , projecting from the framework that supported the pendulum. The planes of both mirrors when at rest were vertical and perpendicular to the plane of vibration of the pendulum.

Projecting downward from M_2 is a fine cambric needle d , which engages at right angles a second similar needle f , attached to the arm a by a short wire e . These needles cross about two centimeters below the axis of M_2 . Calling this distance r and the distance from the crossing of the needles to the pendulum bearings R , it is evident that M_2 will move through R/r times as large an angle as the pendulum. M_2 was so balanced as to cause the needles always to be in light contact.

One meter from the mirrors was placed the flash apparatus. The flash was furnished by an **H**-shaped Geissler tube illuminated by an induction coil whose primary circuit was controlled by the pendulum of the laboratory clock. Immediately above the tube was placed the observing telescope. When the pendulum was at rest the mirrors were so adjusted as to throw images of the capillary part of the tube upon the horizontal cross hair of the telescope. The two images were made to overlap throughout a small distance

for greater ease in determining a coincidence. If now the pendulum were swung at full amplitude the flashes in each mirror would divide into two sets, and these would step along the mirrors in opposite directions. Figs. 2 and 3 represent the typical appearance of the two sets of flashes. Each flash was normally double, the light and heavy parts being due respectively to the closing and opening of the primary circuit by the relay attached to the clock. M_2 , having the greater angular velocity, separated the parts of its flash images and also the successive flashes correspondingly more than

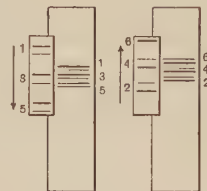


Fig. 2.



Fig. 3.

M_1 . Assuming the flashes to move in the direction indicated by the arrows, if those marked 1 belonging to the downwardly moving set are observed at a given beat of the clock, then those marked 2, belonging to the upwardly moving set, will be seen at the next beat one second later, and so on as numbered.

If the period of the free pendulum be greater than that of the clock, then the distance 1 to 3 represents the amount by which the pendulum fails to reach the position it had two seconds before. Under these conditions the set of flashes in Fig. 2 occur while the pendulum is moving towards the telescope, and those in Fig. 3, while it is moving away. Not more than six or eight consecutive flashes could be observed in M_2 , owing to its shortness and comparatively great angular velocity; the flashes in M_1 , however, remained visible all the time. Flash 7 in M_2 , as figured above, would of course be out of the field of the telescope. At the moment of coincidence the flashes in corresponding sets again form a straight line.

Only those flashes belonging, say, to downwardly moving sets were observed, and only every other coincidence of these, in order to take the coincidences with the two pendulums going in the same direction, and thereby secure from the clock the two-second interval elapsing between two successive passages through the mercury globule in the same direction. The steps of the flashes in M_2 were large even for very small amplitudes of the pendulum. Therefore the coincidences were never ambiguous as they sometimes were with the old arrangement of mirrors.¹ By extending the time of

¹G. R. Putnam, U. S. Coast and Geod. Surv., 1894, app. I.

observation over even so short a period as half an hour, the possible error would appear only in the seventh place of decimals, and even that error would be due to irregularities in the rate of the clock. Usually, however, the pendulum would be started, two or more coincidences observed, and then observations completed on the fork. After an hour or so coincidences were again observed, and the periods of the pendulum and the fork then computed. Each time the quality of the coincidence was noticed and drawn. For accurate work only those coincidences equally good were used. A typical coincidence is shown in Fig. 2, flashes 3. The coincidence was always observed between the heavier parts of the flash images, although the lighter parts had also a valuable function. If, as the flashes moved along the mirrors, the lighter parts *preceded*, then the period of the free pendulum was known to be greater than that of the clock. Now, during an actual observation, if the pendulum's period was greater than one second, and if two successive flash images in M_1 had their lighter parts pointed towards each other, a coincidence was approaching, and the proximity of the two flashes would tell how soon the coincidence was to be expected. Likewise, at the moment of coincidence, all four sets of flashes appearing, the operator had to observe only those flashes whose lighter parts were below, and which consequently belonged to the downwardly moving sets. Thus the flashes labeled themselves, the time spent in actual observation was greatly shortened, and the mind of the operator was left comparatively free at the moment of coincidence, so that the quality of the same could be observed with much greater accuracy.

The energy taken from the pendulum by this set of mirrors was very small, as may readily be seen from the curves of Fig. 4, showing the decrease in amplitude with the time. The upper curve is for the perfectly free pendulum, the lower for the pendulum with both mirror systems in action.

The apparatus for comparing the fork with the pendulum consists of four parts: The flash system, the observing telescope, the starting mechanism, and the arrangements for heating.

1. *The Flash System.*—The periodic short flash of light by means of which the fork was compared stroboscopically with the pendu-

lum, was furnished by a mirror system similar to that described by Reed. In order, however, to reduce as far as possible the friction and moment of inertia of the moving system, a smaller mirror 20 mm. in diameter was used, and the pivot bearings were made from needle points set in shallow glass cups drawn from capillary tubing. The horizontal arm of the rotating mirror crossed the needle projecting from the pendulum, at a distance of five millimeters from the axis of the mirror when the pendulum was at rest.

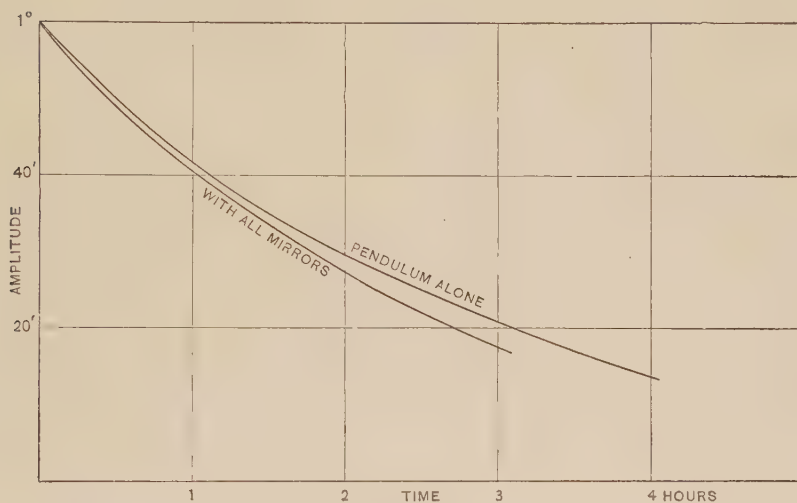


Fig. 4.

The curves of Fig. 4 showing the decrease in the amplitude of the pendulum, with and without the mirror systems, give some idea of the small amount of energy absorbed by the two mirror systems.

Instead of the slit illuminated by a small lamp, as described by Professor Reed, a short straight section of the filament of an incandescent lamp was used. By screening off the light from other portions of the lamp, the flash in the telescope was of sufficient intensity to allow the room to be comfortably lighted during observations.

2 *The Observing Telescope.*—The image of the filament formed by an achromatic lens was thrown by the rotating mirror upon the slit of an Abbé telescope. After passing through a totally reflecting

prism and an adjustable slit in the focal plane of the objective, the light leaves the telescope in parallel rays, falls normally upon the mirror carried upon one prong of the fork to be rated, and returns upon its path to form an image of the slit in the focal plane of the telescope. Fig. 5 shows the arrangement of the optical surfaces

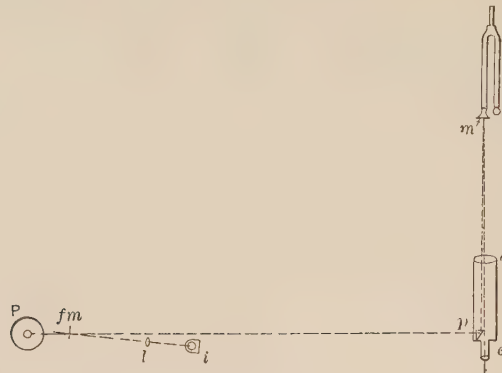


Fig. 5.

and the path of the light from the incandescent lamp i to the observer's eye at e ; l is the projecting lens; fm , the flash mirror; P , the pendulum; m , the fork mirror; p , the reflecting prism; and e , the eye-piece of the observing telescope.

The image of the slit with constant illumination appeared as a broad rectangular band of light when the fork was vibrating, and as a bright slit when the fork was at rest, thus furnishing a means of regulating and measuring the amplitude of the fork. In the focal plane of the eye-piece of the telescope were two cross hairs. One marked the center of the field, and the other, 2 mm. to one side, marked the outer limit of amplitude at which observations on the fork were begun.

3 *The Starting Mechanism.*—Owing to the necessity of enclosing the fork for heating it, some special means of putting it in vibration was necessary. Two methods were used in the preliminary work: One by means of a short wedge thrust sharply between the prongs, the other by means of an electromagnet and an intermittent current from a vibrator in synchronism with the fork. The first method was used only at first, and afterwards kept in reserve as a

check upon the other. The wedge method had two defects: If the fork were already vibrating the wedge would, of course, bring it to rest, while by a skillful use of the second method the operator could add to an already existing amplitude with approximately any desired change of phase. This gave more control over the amplitude at the beginning of an observed cycle. Difficulty would also have been found in applying the wedge method to forks of different dimensions.

In the second method the electromagnet *m* (Fig. 6), arranged to slide back and forth between the prongs of the fork, was connected in series with an outside electric reed capable of being tuned through a considerable range. The reed was a square steel bar, $500 \times 6.4 \times 6.4$ mm., supported on a wooden base by two firm brass uprights with squared holes to fit. The base was supported free from the operating table by two large rubber stoppers, sometimes one at each end and sometimes the rear one moved forward twenty centimeters, according to the method used in tuning the reed. Neither part of the platinum contact was fastened to the reed. The moving part was a thin sheet of platinum, 6×25 mm., one end resting on the reed and the other clamped to a universally adjustable brass pillar. The foil would follow the reed perfectly without interfering with its frequency or with tuning. At the other end of the base was an upright supporting a disk threaded to receive a screw, the head of which was fastened to the end of the reed. By turning the disk the free reed was shortened or lengthened, while vibrating, until the fork would spread. Other methods for obtaining exact unison were frequently used, especially with the more rapidly vibrating forks. One was with the rear supporting stopper moved forward. Then one pressed on the rear of the base, or raised the rear end entirely off of its support and held the base at a suitable angle. It was found that an elevation of 45° would change the frequency by several whole vibrations per second. This change was very gradual at first, thus making the method as sensitive and accurate as one could wish.

The electromagnet *m* and its sliding ways *w* are shown in Fig. 6. The magnet was wound with bare wire, the layers being separated by thin asbestos sheet. One end of the wire was fastened to

the brass magnet spool, thereby making connection through the side wings s , sliding ways, and cast-iron heating shell S , with one end of the circuit of the electric reed. The other end of the magnet winding was fastened to the central draw rod d . This rod was insulated by asbestos from the magnet to which it was fastened, and from the heating shell where it passed out through the front cap c_1 , just below and at one side of the telescope. It served the double function of moving the magnet back and forth on the ways, and of forming the second terminal of the magnet. The circuit, then, included in series the magnet, a key, a resistance box, the electric reed and one or two storage cells.

The key was closed, the reed started, and the magnet drawn forward between the prongs and kept there until the fork was suitably

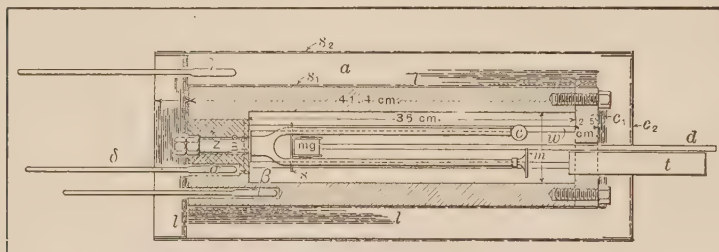


Fig. 6.

spread. Then the key was opened, the magnet pushed back near the base of the fork, and the cycle observed. The dragging effect of the magnet on the frequency of the fork when the circuit was open was closely observed. The maximum effect, when the magnet was at the outer limit, was found to be in the third decimal place. With the magnet half way back the effect was very small, while, when the magnet was pushed back to the limit between the prongs, no effect whatever could be observed.

4. *The Heating Arrangements.*—Fig. 6 shows a longitudinal section through the center of the mounting of the fork. The cast-iron cylinder S , including the cap c_1 , was 44 cm. long, 12.5 cm. outside and 7.5 cm. inside diameter. The side walls were therefore 2.5 cm. thick, the cap was of the same thickness, and the rear end wall had an average thickness of 6.5 cm. The zinc plug Z , in which the fork was mounted so as to insulate it magnetically, was

5.8 cm. in diameter. Such a mass of metal and such large metallic contact with the fork made uniform temperature of the fork easily and quickly obtainable. The total mass of metal to be heated was about twenty-five kilos.

Passing through the cap, immediately in front of the fork mirror, was a heavy-walled brass tube t , 2.5 cm. in diameter and 15 cm. long. The outer end carried a small glass plate set slightly inclined to the plane of the mirror in order to avoid troublesome reflections from its surfaces. Through this tube the observations on the cycle were made. The arrangement of the fork and magnet in the shell is sufficiently clear from the figure.

Surrounding the shell S_1 was a layer of thin asbestos, and over this was wound a layer, of number twenty-eight, tinned, iron wire, seven turns per centimeter. Then came a second layer of asbestos and a layer of wire wound in the reverse direction and connected in parallel with the first layer. The total resistance, cold, was about sixty ohms. Over all was wrapped nine more layers of asbestos. The cap was protected with several layers of asbestos, while the rear wall, where the thermometers were, and where the fork made metallic contact with the shell, was covered with asbestos disks to the depth of four centimeters. The outer shell S_2 was made of sheet iron lined with asbestos and supported 15 cm. above the table on six legs. An air space a , of about four centimeters, was left all around between the two shells, excepting at the rear. The inner shell rested on two curved blocks of wood. In the top of the outer shell were two asbestos-packed openings to admit the terminals of the heating circuit. The head was perforated to admit the observation-tube and the draw-rod. Two thermometer holes were drilled in the rear wall of the heating cylinder. One, α , was in the outer edge of the zinc plug, $1\frac{1}{4}$ cm. in diameter and 5 cm. deep. The other, β , was 2.5 cm. out from this, $1\frac{1}{4} \times 12$ cm., and extended into the side wall. The thermometer bulbs were packed into these holes with tinfoil. The reading of the inner thermometer was taken as the temperature of the fork. Six thermometers in all were used: Two in the shell, α and β , one in the air space, γ , one just outside, δ , near the stem of a , one for the room temperature near by, ϵ , and one near the pendulum, ξ . The thermometer in

the shell wall always read a little higher than that in the zinc plug.

The perfection of the heat insulation was shown by the facts that the temperature of the air space never rose much above 50° C., even when that of the fork was near 200° ; and that thermometer, δ , never read more than 2° or 3° higher than thermometers ϵ and ξ . The hand held near the outer shell felt but little heat, even when working at high temperatures and after the high temperature had been maintained for several hours.

The source of heat was a 220-volt, direct current. The maximum current used was about three amperes. The method of heating was as follows: Suppose it were required to raise the temperature about 10° . The largest current possible was turned on. In from 5 to 20 minutes, after the reading of the thermometers had risen from 5° to 8° , depending on whether the original temperature was low or high, the current was shut off. In 5 to 10 minutes more the temperature would have risen the required 10° and become nearly uniform. Next a comparatively small current, to compensate for radiation losses, was turned on and adjusted frequently during the next half hour or until the readings of the thermometers remained constant, and the constancy of the length of the cycle showed that the fork had assumed the temperature of the zinc plug. As an example of the rate at which the temperature became constant the following table is taken from the record of one day's run:

Time.	Temperature		Amperes.	July 21, 1899.
	α	β		
9:45	38.7°	40°	1.84	Heating current.
9:53	41.6	45.3	off	
10:03	44.42	46.8	0.452	Compensating current.
10:08	44.6	46.6	0.440	
10:13	44.66	46.7	"	
10:21	44.68	46.6	"	
10:32	44.66	46.6	"	

Frequently at untried temperatures the compensating current would have to be changed several times before the temperature could be held constant. Working in this way, under favorable conditions observations could be made at six different temperatures in one day, beginning at 9:00 A. M. and finishing at midnight.

The following extract from the note-book is given as an example of a complete determination :

Time.	Images.	Interval.	Pendulum Amplitude.	Temperature.	The Cycle increases with P , $\therefore c$ is negative.
2:47:43	—	2:40	15 mm.	45.5°	$n = 96$
2:50:23	— —				<i>Fork Cycles</i> 21 + ; 22 — ; 21 + ; 43 ; 65 — $c = -2/43 = -0.046511$
3:22:19	— —	34:36			$N = \frac{95.953489}{1.0129096}$
$\frac{2040}{2014} = 1.0129096 = P$					$= 94.7305$

Time.	Current Amperes.	Temperature.	
		α	β
1 : 56	3.080	23.6°	24°
2 : 12	off	42.2	43
2 : 15	0.620	43.4	47
2 : 22	0.600	44.3	47.4
2 : 27	0.592	44.7	47.5
2 : 37	0.440	45.3	48
2 : 42	"	45.5	"
2 : 52	"	"	47.9
3 : 02	"	"	47.6

The pendulum lost two swings in 2 minutes and 40 seconds. It would therefore lose 26 in 34 minutes and 36 seconds. Hence $\frac{2040}{2014}$ gave the period. The recorded fork cycles were observed just before the last observation on the pendulum. By reference to the table of times and temperatures and from observations in addition to those given above, it was seen that the temperature of the fork, as shown by thermometer α , became constant, 45.5°, at 2.42. The fact that the cycle increased with P was determined before the first pendulum observation by changing slightly the pendulum load, the table showing that the temperature of the fork was then nearly the same as that at which the other observations were made. From previous data n was known to be 96, so that the formula gave $(96 - 2/43)/1.0129096 = 94.7305$ as the frequency of sol_{1a} at 45.5°. Immediately after taking the fork cycle the maximum current

was again turned on and allowed to heat the cylinder to the next desired temperature while the final observations on the pendulum were made, and so on.

To determine the effect of temperature on the damping of the vibrations a third cross-hair was inserted in the eye-piece of the telescope between the other two. The distances from the cross-hairs to the central one were measured by means of a micrometer cathetometer. From the outermost hair to the central one was 2.038 mm.; from the second to the center 0.757 mm. This with the focal length of the objective o , and the distance from the fork mirror to the lens, gave the actual amplitude of the fork. The extreme amplitude for ut_1 was about one millimeter, and that of the other forks was in proportion to their length.

The magnification of the eye-piece was about 13. The fork was spread at the given temperature and the time noted for the edge of the rectangular illumination of the field to fall in from the outermost cross-hair to the second one, and again from the outermost to a point half way between the other two. This latter distance represented the extreme decrease in amplitude permissible in the fork before the cycle flashes would overlap, and the corresponding time was the longest time a set of cycles could run in any one observation. When this period was short the cycle length was adjusted so as to nearly equal it. When it was long the cycle was made some submultiple of it.

PART II. RESULTS OF OBSERVATIONS.

Four forks were used. They formed a set made by König some twenty years ago, of uniform shape and cross-section and differing only in length. They were marked: one, ut_1 , 128 *v. s.*, two sol_1 , 192 *v. s.*, and one ut_2 , 256 *v. s.* The principal dimensions were as follows:

Breadth of prong	14.5 mm.
Thickness of prong	6.0 mm.
Between prongs	26.0 mm.

Lengths from end of prong to inside of curve at base:

ut_1	255 mm.
sol_1	208 mm.
ut_2	175 mm.

When the forks were rated one prong carried a nickel-plated steel mirror 30 mm. in diameter and extending out 16 mm. from the end of the prong. The other prong carried a brass ball as a counterpoise. The mass of ball and of mirror was 20 grams each.

The first fork used was the ut_1 . It was used for all the preliminary work from January 11 to June 20, 1899. This preliminary period included 45 complete determinations of the frequency at temperatures varying from 16.3° to 26.3° ; 22 in the open air, and the remainder with the fork mounted in the cast-iron cylinder but before the addition of any asbestos or wire. In most of the determinations made in the open air the fork was started with a bow. Besides these many partial determinations were made in which the fork was alternately bowed and electrically started. The close agreement of all the results obtained showed that the disturbing effect of the electric starter when used as indicated above, was much smaller than errors due to variations in the rate of the clock.

From the middle of May to the middle of June the heating arrangements were added and perfected. From June 20 to July 1, the 37 determinations with ut_1 recorded in Table I. were made. This table also shows the results of five determinations with one of the sol_1 forks, hereafter to be designated as sol_{1a} .

Table II. shows the effect of temperature upon the damping of the fork. In the first column under each date are given the temperatures; in the columns marked y_1 are the times required for the amplitude to fall from the outer cross-hair to the second one; and in the columns marked y_2 are the times required for the amplitude to fall from the outer cross-hair to a point half way between the other two. The particular fork used on each date is designated at the head of the appropriate column by ut_1 , sol_{1a} , sol_{1b} , and ut_2 respectively. ut_1 and sol_{1a} were the forks whose frequencies were determined.

TABLE I.

Number. Fork ul_1 .	Date, 1899.	Period of Pendulum.	Cycle Length.	Frequency of Fork.	Temper- ature.
1	June 20	1.0074953	16.0	64.4544	37.2
2	" "	1.0084269	14.0	64.3859	47.0
3	" 21	1.0055866	10.0	64.5394	25.1
4	" "	1.0070281	17.6	64.4900	32.4
5	" "	1.1079523	16.25	64.4263	41.2
6	" "	1.0074813	20.5	64.4708	35.35
7	" 22	1.0056054	10.6	64.5438	24.38
8	" "	1.0079545	13.8	64.4151	43.19
9	" "	1.0084122	15.5	64.3939	46.0
10	" "	1.0074836	12.5	64.4377	39.84
11	" "	1.0074627	18.0	64.4633	36.35
12	" "	1.0072522	20.5	64.4835	33.5
13	" 23	1.0055896	9.0	64.5282	26.3
14	" "	1.0074844	15.0	64.4509	38.65
15	" "	1.0084112	11.0	64.3676	49.6
16	" "	1.0093777	13.0	64.3199	54.75
17	" "	1.0098328	14.0	64.2963	57.75
18	" "	1.0102929	13.7	64.2655	61.55
19	" 28	1.0103016	10.0	64.2382	66.25
20	" "	1.0112069	11.6	64.1943	71.55
21	" "	1.0122180	13.5	64.1422	77.35
22	" "	1.0126705	10.5	64.0926	83.3
23	" 29	1.0140575	22.0	64.0541	88.35
24	" "	1.0141271	11.0	64.0048	94.05
25	" "	1.0141079	7.0	63.9548	100.41
26	" "	1.0159680	11.6	63.8945	107.25
27	" 30	1.0170793	12.4	63.8292	114.95
28	" "	1.0055918	7.6	63.7749	121.4
29	" "	1.0055918	15.0	63.7104	128.7
30	" "	1.0070211	10.75	63.6461	135.9
31	" "	1.0079448	11.2	63.5841	142.7
32	" "	1.0088731	12.4	63.5170	150.0
33	July 1	1.0097767	12.8	63.4577	157.0
34	" "	1.0107527	24.5	63.3595	167.3
35	" "	1.0117302	23.0	63.3009	176.3
36	" "	1.0135922	16.0	63.2034	185.8
37	" "	1.0155102	15.0	63.0881	197.0
Fork					
$sol_1(a)$					
1	" 20	1.0102612	31.0	94.9929	23.6
2	" "	1.0129096	21.5	94.7305	45.5
3	" "	1.0159744	11.5	94.4049	75.75
4	" "	1.0097943	15.5	94.0146	109.35
5	" "	1.0152526	20.0	93.5334	148.1

TABLE II.

<i>ut</i> ₁ , July 3.			<i>sol</i> _{1a} , July 5.			<i>sol</i> _{1b} , July 10.			<i>ut</i> ₂ , July 11.		
Temp.	<i>y</i> ₁	<i>y</i> ₂	Temp.	<i>y</i> ₁	<i>y</i> ₂	Temp.	<i>y</i> ₁	<i>y</i> ₂	Temp.	<i>y</i> ₁	<i>y</i> ₂
27.6	31	56	53.0	25	45	22.2	38	66	29.7	28	50
44.5	15	26	68.5	21	35	32.6	34	58	37.2	25	44
52.6	11	20	86.7	18	30	40.3	29	48	34	27	47
63.5	8	14	98	21	36	49.6	23	38	56	18	31
73.5	7	13	76	15	25	63.0	18	31	72.2	13	23
58.4	8½	16	110	21	36	72.5	15	25	85	11	19
88.6	11	19	124.5	24	44	83.3	14½	24½	100	13	23
95.2	14	24	137.5	23	41	95	16	26	122	17½	30
107.3	22	40	147.1	22	38	117.5	17	28	143.5	18	31
116.0	27	50	157.8	21	36	135.6	16	26	166.5	16	28
125.4	32	58	166.2	21	36	149.6	14	22	177	15	26
136.7	33	60	176.1	20	35	159.6	13	20	<i>ut</i> ₁ , Jan. 30		
144.6	34	62	185.8	20	34	173	11	18	to Feb. 9, 1900.		
157.2	35	64	201	19	31	187.8	10	15	—9°	56	98
166.	36	68	212.9	18	30	202.4	10	15½	—2.3	55	96
177.3	33	62	220	18	30	219.5	9½	15	24.33	45	75
186.	32	60	226	17	27	227	9	15	49.1	18	30
196.2	31	58	232	15	25	<i>sol</i> _{1b} , July 12.			83.3	11	18
209.	30	56	242	15	25	23.5	42	70	117.7	10	20
<i>ut</i> ₁ , July 4.			247	15	25	26.6	35	55	152.3	22	44
27.4	32	54	256	15	25	35.6	26	42	188.6	30	52
40.4	19	33	<i>sol</i> _{1a} , July 7.			49	15½	25	232.2	26	46
48.1	14	22	24.4	42	70	62.3	11	18	256.8	28	50
51.0	12	21	35.4	36	60	80	9	14			
64.0	9	16	42.8	30	50	99.3	10	16			
72.8	8	13	51	23	38	119	12	19			
83.5	9	17	64.1	16	27	139.5	12	18			
98.7	15	25	71	13	23	146	10	16			
<i>sol</i> _{1a} , July 20.			83.5	13	22	162	10	16½			
23.6	46	80	89	15	25	184	11½	19			
45.5	30	48	100.5	19	32	200	12	20			
75.7	13	21	114.2	26	42						
109.4	28	48	120	31	51						
148.1	30	51	132.7	33	54						
			143.3	33	54						
			152	31	52						
			163.8	30	50						
			171.6	29	48						
			185.7	26	43						
			210.5	21	35						
			218.5	19	31						

In all cases the fork carried the usual mirror and counterpoise just as when the frequency was being observed, excepting on July 4, 1899. When the phenomena of the effect of temperature on damping was first noticed with ut_1 , and before the temperatures had been reached at which longer time of motion began, it was thought that differences in the loads on the two prongs might be the cause. Accordingly two light loads were made exactly alike except that one was polished to a mirror surface. The values under July 4, 1899, show results practically the same as before.

The values for sol_{1a} are under three dates. On July 5, 1899, the recovery of longer time of motion was not very marked, and other irregularities appear. This was the first time this fork had been heated. But on July 7 and 20, under the second and third heatings respectively, sol_{1a} behaved practically as ut_1 , *i. e.*, with

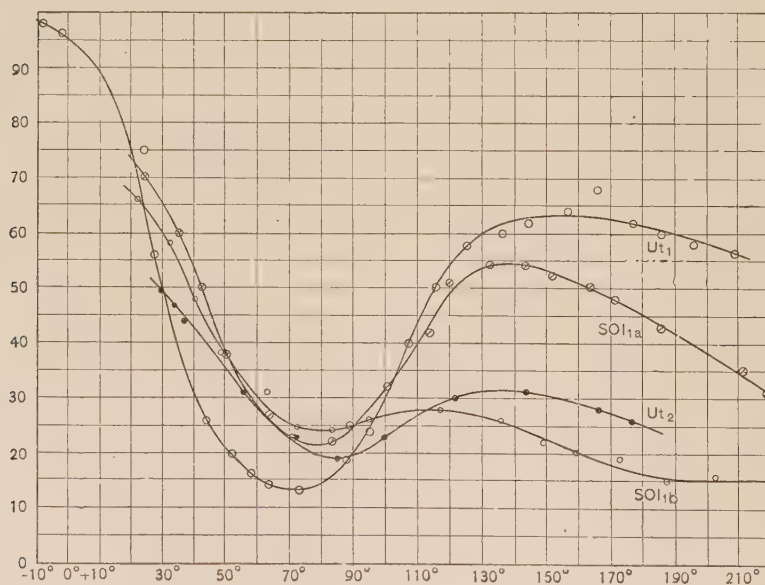


Fig. 7.

a large and continuous decrease in duration of motion followed by almost complete recovery at higher temperatures. July 10 sol_{1b} on being heated for the first time, behaved somewhat as sol_{1a} under similar conditions, but with even smaller recovery. On the second heating, July 12, the decrease was almost as much as that

of ut_1 . The recovery, however, was still very small and irregular, unlike sol_{1a} on its second heating. But a third heating in January, 1900, gave results with sol_{1b} very similar to those with ut_1 . Lastly, ut_2 , when heated for the first time on July 11, showed the same characteristics as ut_1 though in a less degree.

Reference to the curves of Fig. 7 will make these points more evident. The times of motion as given in the columns marked y_2 are plotted as ordinates and the temperatures as abscissæ. The data of July 3 gave the curve marked ut_1 . The second heating of sol_{1a} , July 7 gave the curve marked sol_{1a} , while the first heatings of sol_{1b} and ut_2 on July 10 and 11, respectively, gave the curves labelled sol_{1b} and ut_2 . These latter curves are typical of first heatings, the first heating of sol_{1a} giving very similar results; while the curves labelled ut_1 and sol_{1a} seem to be normal curves, resulting from repeated heatings, and representing, perhaps, a more stable condition of the forks.

To recapitulate: *All the forks agree in having a minimum duration of motion, and this occurs at about the same temperature, $80^{\circ}C$.* Repeated heatings seem to increase the effect, *i. e.*, the decrease in time of motion becomes more marked and the recovery becomes more complete. The phenomenon seems to be entirely independent of the effect of temperature on the frequency, the frequency curves being nearly linear throughout.

Thermometers.—The seven thermometers used were frequently compared. Three of them constituted a set of normal thermometers made by W. Haak, of Jena, September, 1897, and standardized by the *Reichsanstalt* in 1898. They were graduated in fifths of a degree, and had the following ranges and numbers:

No. 11021	+ 20° to + 105°
No. 11022	— 5° “ + 105°
No. 11023	— 90° “ + 210°

These three were used as standards for the others.

Rate of the Clock.—The clock with which the pendulum was compared was a laboratory clock of fair accuracy. When the mercury contact was in action the clock had a losing rate. Compared with the standard Riefler clock in the Physical Laboratory it

was found to be losing quite uniformly one second in ten hours. Therefore to obtain the absolute values of the frequency of nt_1 , $\frac{1}{36000}$ of its apparent frequency should be subtracted therefrom. When its observed frequency was 64.8000 its real frequency was 64.7982. However, the rate of the clock being practically uniform, the relative frequencies would remain as given in the tables. In any case the form of the curves, the real object of the research, would remain unaltered.

Limits of Accuracy. — The method, as finally perfected, eliminated in large measure such errors of observation as might have affected the desired accuracy, *i. e.*, the third or fourth decimal place. The most probable remaining sources of error were: variations in the rate of the clock, uncertainties in determining the length of the cycle and the temperature of the fork. The method of timing the pendulum admitted of no error greater than 0.00001 provided the rate of the clock remained constant. Now to produce an error of that magnitude the rate of the clock would have to vary undetected by one second in ten hours. But such an error in the period of the free pendulum would produce an error of less than one in the third decimal place in the frequency of the fork. There remains one other way, however, in which the period of the pendulum might be in error. Let us suppose the most unfavorable case that might arise in the observation of the last coincidence and also assume conditions under which it would give the greatest error. Suppose the flash in mirror M_2 that was taken as the time of coincidence was as far above the flash in mirror M_1 as the next flash, of the same system two seconds later, was below it, the approximate double-coincidence period to be six minutes, and the last coincidence to be the sixth. Then the true period of the pendulum would be 1.0055834 instead of 1.0055866, as it would have been if the coincidence had been exact. The resulting error, 0.0000032 second, would be much less than the above-mentioned error of one in the fifth place and so much the more negligible. However, by observing the quality of the coincidences, and always testing those for first and last that were alike, this source of error was removed within the limits assigned.

The possible error in reading the thermometers was, from the nature of the instruments used, $\pm 0.025^\circ$. But even here an error

as large as 0.1° would affect only the fourth place in the frequency of the fork. Whether or not the reading of the thermometer α was the exact temperature of the fork could not be determined with absolute certainty. But since the fork and thermometer both made such good metallic contact with the zinc plug, since the reading of the thermometer was kept constant until the length of the stroboscopic cycle between fork and pendulum became constant, and since a variation of 0.1° in the temperature of the fork would ordinarily change the length of the cycle by the readily detected amount of two per cent., it seems reasonable to assume the error in taking the reading of the thermometer as the temperature of the fork to be no greater than $\pm 0.1^{\circ}$ C. This would mean a possible average error of ± 0.0006 in the frequency of ut_1 .

The greatest chances for error, after irregular clock-rate, were in the observation of the fork cycle. This observation required by far the most skill; since accurate results depended to so great a degree on the ready detection and elimination of irregular workings of fork and flash system, and on the degree of fatigue of the eye. The time the flash took to pass the eye-piece slit was calculated to be approximately 0.0001 second. The time could of course be increased or decreased at pleasure, either by changing the amplitude of the pendulum or the length of the needle arm of the flash mirror, or by changing the distance from pendulum to fork. The limits of the room kept this latter distance constant at about four meters. The velocity mentioned, however, was found the best with the forks used so that the image would not be unduly spread out in the field by the motion of the fork. In fact this velocity was so great that very little spreading was observed when the fork was at full amplitude, and none at all when the amplitude had decreased so that the flashes came close together in the field; in which case three successive flashes might fall very near the cross-hair, making exact location of the coincident flash especially desirable. But very little error, then, could arise from this source.

Supposing, however, that a fractional number of flashes formed a cycle; then either the fractional part must be estimated from the position of two successive flashes on either side of the cross-hair, or the counting of the flashes must continue until the fractions have

added up to a whole number. The latter method was used whenever the fork would vibrate long enough without too great a variation in amplitude. But when the duration of motion had decreased with rising temperature to less than twenty-five seconds, the fractions had to be estimated. Suppose that in the case assumed above, where $P = 1.005600$ and $a = 20$, that an error even as large as 0.5 of a flash interval occurred in estimating the fractional part of a . Then if $a = 20$, N would be 64.5883; while if a were 20.5, N would be 64.5895, an error of but 0.001 of a vibration. However, in no case was the possible error in a as large as 0.5. Still, when the temperature was near the minimum for duration of motion the conditions for accurate work would then be the worst possible, and the error might be ± 0.5 of a flash interval if the duration of motion was very small. This would make an error of but one in the third place in decimals. This latter source of error is by far the most serious of the errors of observation. Still, only when the duration of motion becomes inconveniently small does the possible error become of much importance. Ordinarily the error due to this cause would be ± 0.0005 for ut_1 .

Recapitulating the errors due to the various causes:

(1) Irregular rate of clock	± 0.00020
(2) Timing pendulum	± 0.00005
(3) Temperature of fork	± 0.00060
(4) Error due to cycle	± 0.00050

These are probable errors when working above 100° . Between 50° and 100° error (4) might be larger. Below 50° errors (3) and (4) could be made much smaller. The total probable error then, under ordinary conditions, would be given by $\epsilon = \{ (20^2 + 5^2 + 60^2 + 50^2) \times 10^{-5} \}^{\frac{1}{2}} = \pm 0.0008$. If necessary all the errors could be limited to the fifth place in decimals for determinations near the temperature of the room.

When short-cycle lengths were used the sensitiveness of the method was sufficient to show distinct variations in the frequency in different parts of even the limited range of fork amplitude used. Also as the pendulum ran down in amplitude, the change in cycle length would show the decrease in period of the pendulum.

PART III. APPLICATION OF RESULTS.

The results in Table I. are embodied in the curves of Fig. 8. Temperatures are plotted as abscissæ and frequencies as ordinates. The curves, it will be noticed, deviate in an unmistakable way from linearity. The change in curvature per degree seems to be approximately constant. Hence $\frac{d^2N}{dt^2} = \text{a constant}$ within the limits of observation. König, working between 20° and 50° , de-

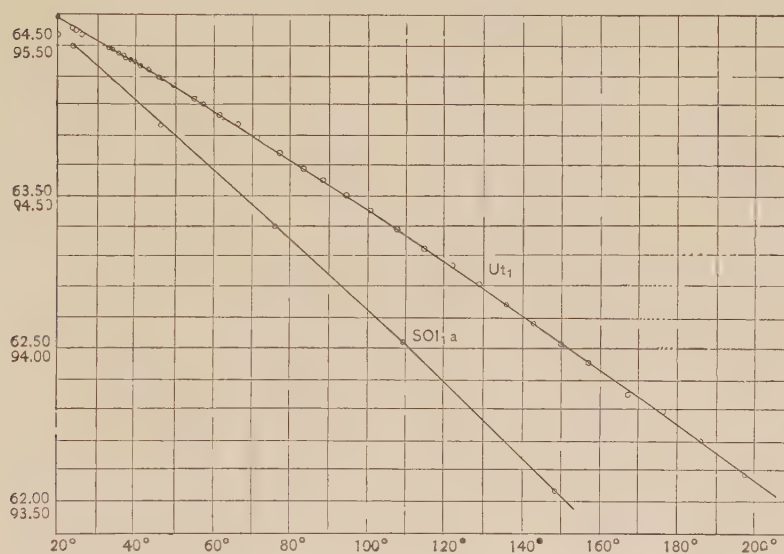


Fig. 8.

cided that the decrease in frequency decreased with higher temperatures. The above curves, however, seem to indicate that this decrease in frequency constantly *increases* with rising temperature.

Another important characteristic of the curves is that the one for sol_{1a} is much flatter than that for ut_1 . The reason for this will appear when the fundamental formulæ involving relative frequencies at different temperatures are considered.

Kayser (also Mercadier) gives the formula for the frequency of a fork as follows :

$$(i) \quad N_0 = \frac{e_0 k}{l_0^2 \sqrt{D_0 \gamma_0}},$$

where k is a constant, N_0 the frequency, e_0 the thickness and l_0 the length of the prongs, D_0 the density, and γ_0 the coefficient of elasticity, all at 0° C. Parenthetically it should be stated that this elastic coefficient of Kayser, and of Wertheim, is the reciprocal of the coefficient of Mercadier, Miss Noyes and others. Therefore this coefficient will increase with rising temperature where they find their's to decrease.

The frequency at any other temperature will be given by :

$$(ii) \quad N_t = \frac{e_0(1+at)k(1+at)^{\frac{3}{2}}}{l_0^2(1+at)^2\sqrt{D_0\gamma_t}} = \left\{ k \frac{e_0}{l_0^2\sqrt{D_0}} \right\} \left(\frac{1+at}{\gamma_t} \right)^{\frac{1}{2}},$$

where a is the coefficient of expansion of steel, and γ_t is the elastic coefficient at t° .

Combining (i) and (ii) we obtain

$$(iii) \quad \frac{N_t}{N_0} = \left\{ \frac{\gamma_0}{\gamma_t}(1+at) \right\}^{\frac{1}{2}} \text{ or } N_t = N_0 \left\{ (1+at) \frac{\gamma_0}{\gamma_t} \right\}^{\frac{1}{2}}$$

and

$$(iv) \quad \frac{\gamma_t}{\gamma_0} = \left(\frac{N_0}{N_t} \right)^2 (1+at) \text{ or } \gamma_t = \gamma_0(1+at) \left(\frac{N_0}{N_t} \right)^{\frac{1}{2}}.$$

Now it will be seen from (ii), (iii) and (iv), that

1. If γ were constant N_t would be greater than N_0 . Therefore
2. The effect of temperature on the dimensions of the fork opposes that on the elasticity, as far as concerns the frequency ; and
3. The change in frequency due to the effect of temperature on the elasticity is the greater of the two, since $N_0 > N_t$.
4. Neither effect is linear.
5. The total effect is not linear but increases with the temperature.
6. The effect of temperature on γ increases with the temperature.
7. The total effect is proportional to the first power of N .
8. Since the dimensions of the fork appear as constants in (ii), the coefficient of the total effect is independent of the ratio of e to l , *i. e.*, independent of the dimensions of the fork.
9. γ_t not being a linear function, the flatter frequency curve for sol_{1a} may be due to a flatter γ_t curve for the steel of which the fork is made. (Neglecting the fact that a is only approximately linear.)
10. N_t would be linear only if the temperature coefficients of expansion and elasticity were equal.

Kayser says that the total effect is linear, from 0° to 30° , and that the effect is proportional to the square root of the frequency.

König finds the total effect to be non-linear, and that it is proportional to the frequency but that the coefficient decreases with rising temperature. Pierpaoli reports results in these respects similar to those of König. The above ten conclusions from the formulæ universally accepted agree with similar conclusions from the data accumulated in this research.

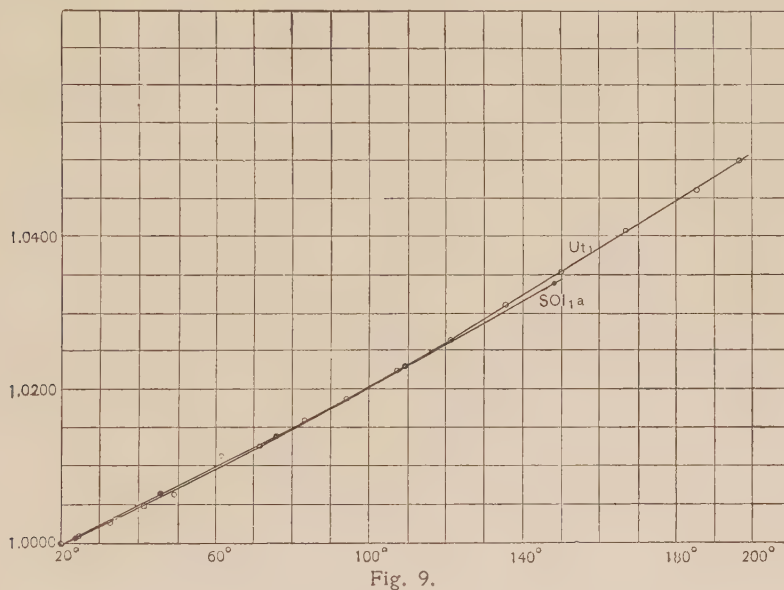


Fig. 9.

The curves in Fig. 9 represent the variations of γ with temperature. In each case γ_{20} was taken as unity and the ratio $\frac{\gamma_t}{\gamma_{20}}$ calculated by means of formula (iv) and the data in Table I. Table III. gives the calculated ratios.

The curves differ slightly for the two forks. Both were probably from the same steel but had doubtless passed through experiences sufficiently different to have changed γ permanently by a small amount. Both however show decided convexity towards the axis of temperatures, upholding conclusion (6). Still the departure from linearity is small, and especially since γ_t enters equation (iii) under the radical, the error in N_t made by assuming γ_t linear would

TABLE III.

Temperature.	$ut_1, \gamma_t + \gamma_{20}$	Temperature.	$sol_{1a}, \gamma_t + \gamma_{20}$
20.00	1.0000	20.00	1.0000
24.38	1.0010	23.60	1.0008
32.40	1.0028	45.50	1.0066
41.20	1.0048	75.75	1.0139
49.60	1.0065	109.35	1.0227
61.55	1.0101	148.10	1.0337
71.55	1.0125		
83.30	1.0158		
94.05	1.0188		
107.25	1.0224		
121.40	1.0264		
135.90	1.0307		
150.00	1.0351		
167.30	1.0404		
185.80	1.0458		
197.00	1.0498		

be very small for a range of 40° . Accordingly formula (iii) may be written :

$$(v) \quad N_t = N_{20} \left\{ \frac{1 + \alpha (t - 20)}{1 + \beta (t - 20)} \right\}^{\frac{1}{2}}, \text{ where } \beta = \frac{\gamma_t - \gamma_{20}}{t - 20}$$

This expression differs in form from those usually given ; such as :

$$N_t = N_0(1 - at) \text{ or } N_t = N_0(1 - at + bt^2);$$

and is applicable not only to the forks experimented with, but is equally true for all forks, given the two coefficients of the material of the fork. The only assumption made is that the two coefficients, α and β , are constant over a desired small range of temperature. However for larger ranges and more accurate results, the fact that γ_t is not linear must be taken into account. Then the formula becomes

$$(vi) \quad N_t = N_{20} \left\{ \frac{1 + \alpha (t - 20)}{1 + \beta (t - 20) + \delta (t - 20)^2} \right\}^{\frac{1}{2}}$$

The values calculated for ut_1 were :

$$\alpha = 0.0000112 \text{ (Kayser)}$$

$$\beta = 0.00023$$

$$\delta = 0.0000003$$

and for sol_{1a} .

$$\beta = 0.000238$$

$$\delta = 0.00000019$$

Table IV. gives the data from which these results were obtained and also the computed values of N_t for sol_{1a} , using (vi). The computed values for ut_1 were obtained by using the conventional formula :

$$N_t = N_{20} \{ 1 - a (t - 20) - \beta (t - 20)^2 \}$$

a and β were found to be :

$$a = 0.000108$$

$$\beta = 0.00000014$$

This expression of the second degree in t is not sufficient for the range of temperature covered. The computed values at the higher temperatures show a too rapid decrease. If greater accuracy over the whole range of temperature were desired, terms in values of higher powers of t would be needed. Therefore the accuracy of the observed values in Table IV. is not to be judged by the magnitude of the values in the column of differences, but rather by the exactness with which the differences themselves approximate a law of increase and decrease, changing sign at 50° and 150° and varying with more or less regularity.

As before mentioned the coefficient of elasticity as used above is the reciprocal of that of Mercadier. Taking the reciprocals of the above ratio values at 20° and 120° , we see that the coefficient, as used by Mercadier, decreases for the 100° from 20° to 120° by 2.53 per cent. Mayer¹ found that heating from 0° to 100° lowered the modulus of cast steel 2.21 per cent., and Bessemer steel 3.08 per cent. His values were obtained by observing the frequency of vibrating bars supported at the nodes. Miss Noyes, obtaining Young's modulus of a wire by stretching, found an average decrease of 4.6 per cent. per 100° . Wertheim for 100° found about 2.8 per cent., also using a stretched wire, while Kohlrausch and Loomis, working with a torsionally vibrating wire of long period, found an average decrease of 4.6 per cent. per 100° . Thus the per cent. change for 100° of the temperature coefficient

¹ Rep. of Brit. Assoc., 573, 1894.

TABLE IV.

Temperature.	Frequency of ut_1 .			Temperature.	Frequency of sol_{1a} .		
	Observed.	Computed.	Excess of Observed.		Observed.	Computed.	Diff.
20.00	64.5760			20.00	95.0300		
24.38	64.5438	64.5452	- 14	23.60	94.9929	94.9910	+ 19
25.10	64.5394	64.5401	- 7	45.50	94.7305	94.7493	-188
26.30	64.5282	64.5317	- 35	75.75	94.4049	94.4075	- 26
32.40	64.4900	64.4880	+ 20	109.35	94.0146	94.0116	+ 30
33.50	64.4835	64.4801	+ 33	148.10	93.5334	93.5393	- 59
35.35	64.4708	64.4668	+ 40				
36.35	64.4633	64.4595	+ 38				
37.20	64.4544	64.4533	+ 11				
38.65	64.4509	64.4427	+ 82				
39.84	64.4377	64.4340	+ 36				
41.20	64.4263	64.4240	+ 23				
43.19	64.4151	64.4093	+ 57				
46.00	64.3939	64.3885	+ 54				
47.00	64.3859	64.3810	+ 49				
49.60	64.3676	64.3616	+ 60				
54.75	64.3199	64.3227	- 28				
57.75	64.2963	64.2998	- 35				
61.55	64.2655	64.2706	- 51				
66.25	64.2382	64.2341	+ 41				
71.55	64.1943	64.1924	+ 19				
77.35	64.1422	64.1463	- 41				
83.30	64.0926	64.0982	- 56				
88.35	64.0541	64.0571	- 30				
94.05	64.0048	64.0099	- 52				
100.41	63.9548	63.9567	- 19				
107.25	63.8945	63.8987	- 42				
114.95	63.8292	63.8323	- 31				
121.40	63.7749	63.7758	- 7				
128.70	63.7104	63.7110	- 6				
135.90	63.6460	63.6462	- 1				
142.70	63.5841	63.5841	0				
150.00	63.5170	63.5165	+ 5				
157.00	63.4577	63.4508	+ 69				
167.30	63.3595	63.3525	+ 70				
176.30	63.3009	63.2650	+359				
185.80	63.2034	63.1710	+324				
197.00	63.0881	63.0583	+298				

of elasticity seems to vary within quite wide limits with variations in the quality of the steel, the form in which it is used, and the method by which the coefficient is determined. The manipulations

in obtaining the different forms, and in the after experimenting practically amount to changing the quality of the material. For this reason, work with a wire would hardly be a criterion for bars, and *vice versa*, even if originally made from the same quality of steel. Perhaps experiments with bars and forks would have the advantage of greater permanence of the qualities of the material to be investigated, during the experimenting.

The fact remains, however, that the change of the coefficient of elasticity with temperature for several typical kinds and forms of steel, varied only from 2.5 per cent. to 4.5 per cent. for a change of temperature of 100° .

To sum up the chief results of this research :

1. The temperature coefficient for the frequency of a fork is not linear.

(a) The change in frequency is proportional to the first power of the frequency.

(b) The coefficient is independent of the dimensions of the fork.

(c) The coefficient depends only on the coefficient of expansion, and the variation of the coefficient of elasticity with temperature.

2. The coefficient of elasticity of steel has neither a maximum nor a minimum between 100° and 200° , but varies continuously and nearly linearly.

3. A change in temperature affects the logarithmic decrement of a tuning fork in a marked degree. The decrement has a maximum at about 80° C. with minima at about 20° C. and 140° C.

In conclusion the author desires to express his thanks to Professor John O. Reed, for his interest and assistance throughout the progress of this investigation.

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